Large neutrino mixing angles for type-I see-saw mechanism in SO(10) GUT

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We consider the neutrino mixing angles in an SO(10) GUT with the usual Higgs structure in which neutrino masses are explained by the type-I see-saw mechanism. The Dirac-neutrino Yukawa matrix then has a structure similar to that of the u-quark. We determine the light neutrino mass matrix through type-I see-saw mechanism using the experimentally consistent u-quark Yukawa matrix. We find that large neutrino mixing-sngles emerge naturally in this model.

The most natural extensions of the standard model are the grand unified theories, in which the strong, the weak and the electromagnetic gauge coupling constants are unified. Quarks and leptons also becomes part of one single representation of the grand unified group and hence the quark and lepton mass matrices become related. While this makes the model predictive, the largeness of the neutrino mixing angles becomes difficult to explain keeping the quark mixing angles small.

We consider here an SO(10) GUT with standard Higgs structure [1] and left-right symmetric extension [2] of the standard electroweak interaction up to an intermediate symmetry breaking scale. The quark and lepton Dirac masses come from the vacuum expectation value (*vev*) of a bi-doublet Higgs, while the neutrino masses come from the *vev* of triplet Higgs scalars. The left-handed neutrino mass now comes from two sources. There is a *direct* contribution to the left-handed neutrino mass coming from the *vev* of the left-handed Higgs triplet scalar [3]. This is also called a type-II see-saw mechanism, since the *vev* of the left-handed triplet Higgs is see-saw suppressed by the lepton number violating scale. There is also the usual *see-saw* contribution to the left-handed neutrino Majorana mass, which is suppressed by the right-handed neutrino mass (this is also called the type-I see-saw mechanism) [4].

In SO(10) GUTs the Dirac masses of the quarks and leptons come from the same Yukawa coupling. As a result one expects that the mixing angles for both the quark and lepton sectors are similar. Thus, a small quark mixing angle makes it difficult to accommodate a large neutrino mixing angle in the case of see-saw neutrino mass. However, in case of type-II see-saw neutrino mass it is possible to make the neutrino mixing angles large in a supersymmetric SO(10) GUT when the relation $m_b = m_\tau$ is assumed [5].

In this article we point out that in the case of type-I see-saw neutrino mass it is possible to make the neutrino mixing angle large still keeping the quark mixing angles small, while at the same time predict $m_b = m_\tau$.

We shall consider an effective scenario, which may emerge from a supersymmetric SO(10) GUT. The fermions belong to the fundamental representation **16**-plet of Higgs, which transforms under the Pati-Salam subgroup $(G_{422} \equiv SU(4)_c \times SU(2)_L \times SU(2)_R)$ as,

$$\psi_{iL} \equiv \mathbf{16} = (\mathbf{4}, \mathbf{2}, \mathbf{1}) + (\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2}).$$

$i = 1, 2, 3$ is the generation index. The right-handed fermions (ψ_{iR}) then belong to the conjugate representation,

$$\psi_{iR} \equiv \bar{\mathbf{16}} = (\bar{\mathbf{4}}, \mathbf{2}, \mathbf{1}) + (\mathbf{4}, \mathbf{1}, \mathbf{2}).$$

The generators of the left-right symemtric group $G_{3221} \equiv SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ are related to the electric charge by

$$Q = T_{3L} + T_{3R} + \frac{(B-L)}{2} = T_{3L} + \frac{Y}{2}.$$

The quarks and leptons belong to the representations

$$(\mathbf{4}, \mathbf{2}, \mathbf{1}) = \begin{cases} q_L = (u \quad d)_L \equiv (3, 2, 1, 1/3) \\ \ell_L = (\nu \quad e)_L \equiv (1, 2, 1, -1) \end{cases}$$

$$(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2}) = \begin{cases} q_R{}^c = q^c{}_L = (d^c \quad u^c)_L \equiv (\bar{3}, 1, 2, -1/3) \\ \ell_R{}^c = \ell^c{}_L = (e^c \quad \nu^c)_L \equiv (1, 1, 2, 1) \end{cases} \tag{1}$$

where (x,y,z) and (x,y,z,w) denote the transformation property under G_{422} and G_{3221} respectively.

The Higgs scalars which are responsible for the electroweak symmetry breaking and the fermion masses belong to the representations **10** (H) and **126** (Δ). The *vev*s of the right-handed triplets $\Delta_R \equiv (1, 1, 3, -2)$ give Majorana masses to the right-handed neutrinos and the *vev*s of the left-handed triplets $\Delta_L \equiv (1, 3, 1, -2)$ give tiny Majorana masses to the left-handed neutrinos directly. The *vev*s of bi-doublet H give Dirac masses to all fermions. The effective Yukawa couplings are

$$\begin{aligned} \mathcal{L}_Y &= Y_u \overline{q_L} u_R H + Y_d \overline{q_L} d_R H^\dagger + Y_n \overline{\ell_L} \nu_R H \\ &+ Y_e \overline{\ell_L} e_R H^\dagger + f_L \nu_L \nu_L \Delta_L + f_R \nu_R \nu_R \Delta_R. \end{aligned} \tag{2}$$

The left-handed light neutrino mass matrix is then given by

$$M_\nu = M_L + M_D^T M_R^{-1} M_D = M_L + \kappa$$

where the first term is the direct neutrino mass coming from the vevs of the neutral component of the triplet Higgs scalar Δ_L. Since the vev of the scalar Δ_L acquires a see-saw value of the order of $<H>^2 / <\Delta_R>$, this is also called the type-II see-saw mechanism. The second term is the usual or the type-I see-saw neutrino mass. We adopt the notation $M_D = f <H>$, $M_L = f_L <\Delta_L>$ and $M_R = f_R <\Delta_R>$. We shall study the scenario with $M_L < \kappa$, i.e., the usual type-I see-saw neutrino mass dominates over the type-II see-saw neutrino mass.

It is wellknown that since the Higgs scalar (H) is in a 10-plet of SO(10) with fermions in the 16-plet, the structure of the Dirac-neutrino mass matrix will be the same as that of the u-quark [5, 6]. The structure of the u-quark mass matrix at low energies has been studied extensively [7] and the following is obtained for the corresponding Yukawa matrix, Y_u,

$$Y_u \approx \begin{pmatrix} \lambda^7 & \lambda^6 & \lambda^6 \\ \lambda^6 & \lambda^4 & \lambda^4 \\ \lambda^6 & \lambda^4 & 1 \end{pmatrix} m_t \qquad (3)$$

where m_t is the top-quark mass. The above matrix is usually written in terms of a parameter ϵ (≈ 0.05), but we have expressed it in terms of the more familiar, Cabbibo parameter λ ($\approx .22$), utilizing the simple, numerical relation $\epsilon \approx \lambda^2$. A texture zero is also usually inserted for the 11-element of Y_u, however, we have put a non-zero but very small value for this matrix element so that the ratio of the diagonal terms manifestly reproduce the ratio of the up-quark masses. Furthermore it is known that, because of very small mixing angles in the quark-system and very strong mass hierarchy, the scale evolution of Y_u will be negligible [8], so we will assume the above form to be valid at the GUT scale as well. From the SO(10) properties mentioned earlier we will then have, for the Dirac-neutrino,

$$M_D \approx \begin{pmatrix} \lambda^7 & \lambda^6 & \lambda^6 \\ \lambda^6 & \lambda^4 & \lambda^4 \\ \lambda^6 & \lambda^4 & 1 \end{pmatrix} m_3 \qquad (4)$$

where m_3 is the value of the 33-component.

For simplicity, we will take the right-handed Majorana neutrinos mass matrix to be diagonal,

$$M_R = \begin{pmatrix} M_{11} & 0 & 0 \\ 0 & M_{22} & 0 \\ 0 & 0 & M_{33} \end{pmatrix}. \qquad (5)$$

If in addition we assume that this matrix is hierarchical so that $M_{11} \ll M_{22} \ll M_{33}$ then we can, to a good approximation, neglect the contribution of M_{22} and M_{33} in the type-I see-saw relation to obtain the light neutrino mass matrix as

$$\kappa = M_{\nu D}^T M_R^{-1} M_{\nu D} = \left(\frac{m_3^2}{M_{11}} \right) \lambda^{12} \begin{pmatrix} \lambda^2 & \lambda & \lambda \\ \lambda & 1 & 1 \\ \lambda & 1 & 1 \end{pmatrix}. \qquad (6)$$

This matrix shows a maximal angle in the atmospheric neutrino sector. If we allow for a small correction of order λ in the 22-element of κ which could be included, for example, by retaining the matrix elements of M_R in addition to just the 11-element, we would obtain

$$\kappa = \left(\frac{m_3^2}{M_{11}} \right) \lambda^{12} \begin{pmatrix} \lambda^2 & \lambda & \lambda \\ \lambda & 1+\lambda & 1 \\ \lambda & 1 & 1 \end{pmatrix}. \qquad (7)$$

This matrix will reproduce not only the near maximal atmospheric angle but also the large solar and a small angle in the 13-sector [6, 9].

As for the neutrino masses, the eigenvalues of the above mass matrix are $(0, \lambda, 2)$, apart from a common multiplicative constant, which leads to normal hierarchy, $m_1 \ll m_2 \ll m_3$, and gives

$$\frac{\Delta m_{12}^2}{\Delta_{23}^2} \approx \frac{\lambda^2}{4} = 10^{-2}$$

which is consistent with the data. Furthermore, because of the absence of quasi-degeneracy in the masses, large renormalization group effects in the neutrino sector are not expected so that the large mixing angles at the GUT scale indicated by the mass matrix in (7) will stay large at low energies [10, 11].

We further note that because H in our formalism is a 10-plet, it predicts $m_b = m_\tau$, which is consistent with experiments [5]. The remaining matrix elements in Y_e and Y_d are quite small ($\leq \lambda^2$) compared to these masses and any discrepancy between them can be explained in terms of a small contribution of a 126-plet Δ and by introducing appropriate texture zeroes in Y_e and Y_d [11, 12].

We thus conclude that large neutrino mixing angles are feasible, and are compatible with small quark mixing angles, in the type-I seesaw mechanism in a certain class of SO(10) GUT model. We demonstrated this with a specific realistic example.

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